Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 4, 1998

OPTICAL RESPONSE OF A LAYER OF A NEMATIC LIQUID CRYSTAL TO AN AIR FLOW

V. M. Korotaev, S. I. Trashkeev, and G.M. Zharkova

UDC 532.783

A liquid crystal coating is used to measure the surface friction created when a flat plate is subjected to an air flow. Surface friction is determined from the optical response of the nematic liquid crystal coating to the flow. The proposed method does not require precise monitoring of the thickness of the coating or the angles of illumination and observation. This makes it possible to eventually progress to panoramic measurements of surface aerodynamic characteristics.

Attempts to use liquid crystals (LCs) in aerodynamic studies that require visualization and measurement of different characteristics of the interaction of an air flow with a surface have been undertaken for a rather long time [1, 2]. Among them are experimental studies of heat exchange between the flow and the surface and conditions of laminar–turbulent transition, measurements of surface friction, etc. Investigators (see the bibliography in [3]) have focused mainly on the use of cholesteric liquid crystals (CLCs) for these applications, probably because the optical response of these crystals — the change in color due to the air flow — is visible even to the naked eye.

However, the color of the light reflected by a layer of CLC is also affected by several other factors, such as temperature, angle of observation, variation of the layer thickness during the experiment, etc. To obtain a detailed picture of the air flow over a surface under these conditions and measure certain parameters of the interaction with acceptable accuracy, it is necessary to introduce corrections for the effect of the above factors or to compensate this effect by complicating the recording technology [3, 4] and thereby losing the main advantage of the LC-coating technique — the simplicity of implementation. Thus, it is important to devise a method for obtaining a detailed picture of flow over a surface that is simple to use but is also sufficiently informative.

Little attention has been given to the use of nematic liquid crystals (NLCs), although their optical parameters are sensitive to mechanical deformation. Because of the simpler internal structure of this type of liquid crystal, the optical response of NLCs to certain types of deformation can be calculated fairly simply. In particular, Zharkova and Trashkeev [5] established a relationship between the optical response of an NLC layer and mechanical parameters of its shear flow under the influence of an air flow.

In the present investigation, we showed experimentally that the optical response of an NLC layer to an air flow within a certain velocity interval unambiguously characterizes the shear stress on a surface exposed to the flow. The geometry of the interaction was the same as in [5], the only difference being that recording of the response did not require radiographic inspection of the NLC layer. This is especially important for aerodynamic applications. In contrast to [4], where a CLC was used, no complicated equipment such as a spectrum analyzer is required.

Of course, many of the shortcomings inherent in the LC-coating technique remain. Nevertheless, there are methods of surmounting some of these problems, such as the dependence on the angle of observation and the effect of the coating thickness.

Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 4, pp. 102–109, July–August, 1998. Original article submitted December 30, 1996.

0021-8944/98/3904-0569 $20.00 © 1999 Kluwer Academic/Plenum Publishers

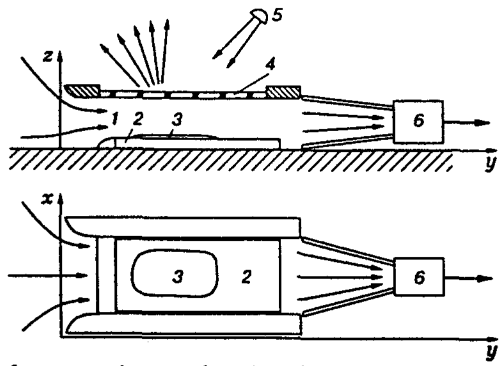

Fig. 1. Diagram of the experimental unit: 1) working part (channel of rectangular cross section); 2) flat plate; 3) NLC "spot;" 4) transparent window; 5) light source; 6) flow regulator.

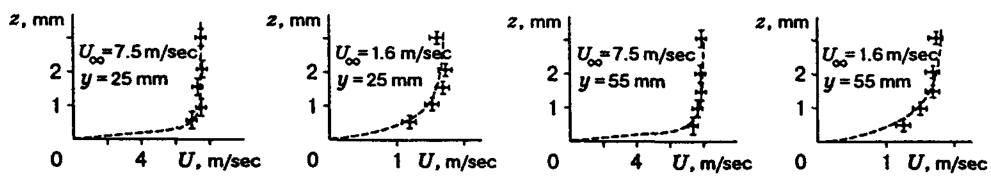

Fig. 2. Flow-velocity profiles measured above the plate at two points for two flow velocities.

Figure 1 shows a diagram of the setup used in the investigation. Air flow was produced in the working part 1 (a rectangular channel with a cross section of 8 × 25 mm). The velocity of the flow was established by means of flow regulator 6. Figure 2 shows the velocity profiles of the flow above the plate in two sections of the channel: at the beginning of the working section ($y = 25$ mm) and at the end ($y = 55$ mm). The object of study — a flat plate 2 coated with a thin NLC layer 3 — was placed on the bottom wall of the channel. The object was illuminated by plane–parallel light 5 that passed through the transparent top wall 4. The scattered light was directed onto a photodetector by a lens.

The object tested was prepared as follows. A flat steel plate with a finish of class 6 or 7 (which corresponds to the actual finish from the machining of aerodynamic models) was coated with a thin polyvinylacetate layer several microns thick, on which the orienting properties for the LC were then produced. A small amount of the nematic liquid crystal in the form of a 4 × 15 mm spot with a depth of 20–30 μm (Fig. 3a) was placed in the middle of the plate. The test NLC layer was shaped by an air flow that was started several minutes before the beginning of the experiment (Fig. 3b). The orientation of the director on the plate was perpendicular to the plane of the figure. The thickness of the layer of the liquid crystal in the region of observation was of the order of 10 μm and decreased during the experiment. The reverse flow of the NLC to its initial position (Fig. 3a) due to surface tension lasted 8 h.

We used nematic liquid crystal mixture 1282, consisting of alkoxycyanbiphenyls. The mixture remains in the liquid-crystal state within a broad range of temperatures (roughly 40°C).

The optical system used in the experiment is shown in Fig. 4. Linearly polarized light from the source 5 illuminates the test surface 2 covered by the NLC layer. The scattered light passes through polarization filter 7 and is directed by lens 8 onto photodetector 9. The region of observation was determined by the diameter of the photodetector aperture, which was 2 mm. Figure 4 shows the direction of polarization of the light source, the direction of the air flow and polarization filter, and the orientation of the liquid crystal on the substrate.

The plate was placed in the channel in such a way that the coordinate of the observation point was 25 mm. We periodically turned on the air flow for a short period of time (10–30 sec). The desired flow velocity was established by the flow regulator in the pauses between the periods when air was flowing.

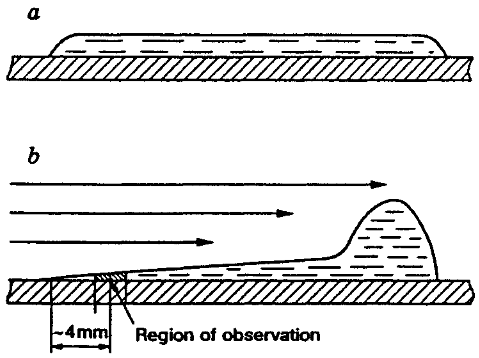

Fig. 3

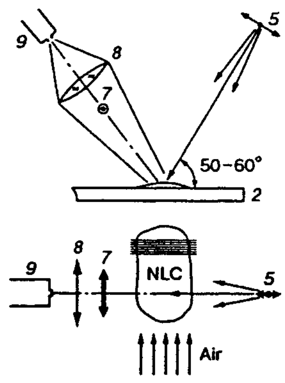

Fig. 4

Fig. 3. Cross section of NLC layer before (a) and after (b) beginning of air flow (the boundaries of the liquid are depicted schematically).

Fig. 4. Optical measurement system: 7) polarization filter; 8) lens; 9) photodetector; the remaining notation is the same as in Fig. 1; the orientation of the NLC is indicated by the horizontal lines.

In the absence of air flow, the orientation of the director in the NLC layer is determined by the substrate, i.e., the director is parallel to the plane of polarization of the incident light. Thus, a significant portion of the reflected light maintains the polarization of the incident light (except for the light scattered due to thermal fluctuations). Only light that does not maintain the initial polarization passes through the polarization filter. The air flow, in interacting with the surface, changes the orientation of the director inside the NLC and brings it closer to a direction parallel to the flow and the orientation of the filter. The polarization of the scattered light becomes such that the light passes through the filter. As a result, there is a sharp increase in the strength of the signal from the photodetector after the air flow is turned on. The character of the change in the signal at different velocities U_∞ is shown in Fig. 5.

The optical response to the flow was calculated as the ratio of the signal in the presence of an air flow H to the signal in the absence of a flow h. Figure 6 shows the behavior of H/h at a constant flow velocity for two objects.

This experiment shows that, despite the decrease in H and h, the ratio H/h is roughly constant over a relatively long period of time. This period (the interval $t_1 - t_2$ in Fig. 6) is equal to 180 sec.

The fact that H/h does not depend on the thickness of the NLC layer indicates that the relatively thin surface section makes the main contribution to the change in the signal. The signal H/h will be roughly constant until the thickness of the NLC layer is comparable to the thickness of the layer that forms the response.

The independence of H/h relative to the thickness of the NLC layer makes it possible to study the dependence of H/h on the velocity of the air flow. Figure 5 shows the experimental results recorded while the velocity of the air flow was varied. The empirical dependence of H/h on the flow velocity U_∞ (Fig. 7) is approximated well by a straight line:

$$H/h = 1 + CU_\infty^{3/2}, \qquad (1)$$

where C is a constant that does not depend on U_∞.

As is known [6], the dependence of surface tension on the velocity of an air flow above a flat plate is expressed by the formula $\tau \sim U_\infty^2/(\mathrm{Re}_y)^{1/2}$. Since $\mathrm{Re}_y = U_\infty y/\nu$ (where y is the coordinate of the observation

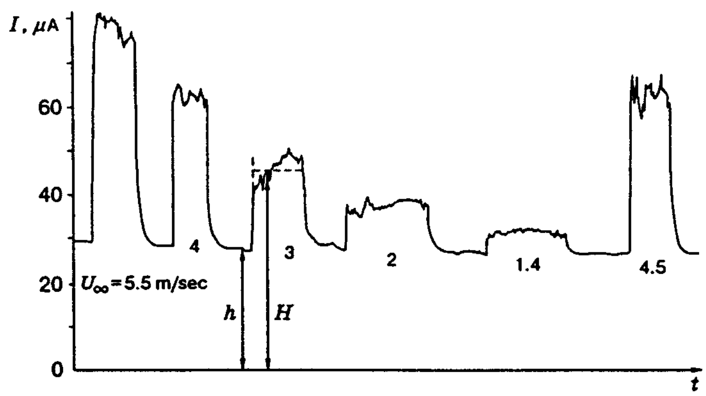

Fig. 5. Signal from the photodetector I in the presence (H) and absence (h) of air flow.

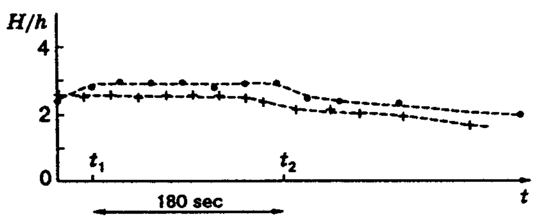

Fig. 6

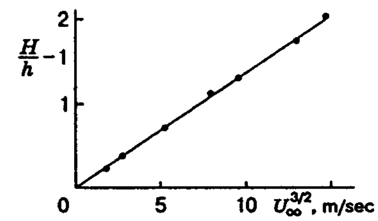

Fig. 7

Fig. 6. Behavior of the optical response H/h versus time at a constant velocity of the incident flow $U_\infty = 5.5$ m/sec.

Fig. 7. Dependence of the optical response $H/h - 1$ on the velocity of the incident flow U_∞ to the power 3/2.

point and ν is the kinematic viscosity of air), we obtain

$$\tau \sim U_\infty^{3/2}. \tag{2}$$

Thus, it follows from (1) and (2) that the parameter $H/h - 1$ is an index of the shear stress τ in the flow of air over a surface:

$$\tau \sim H/h - 1. \tag{3}$$

A mathematical model of the flow being discussed was proposed in [5]. The optical response of an NLC layer to aerodynamic loading was determined by solution of truncated Maxwell equations of the form

$$\frac{d\mathbf{A}}{dz} = iq_a\mathbf{n}(\mathbf{n}\mathbf{A}). \tag{4}$$

Here $\mathbf{A} = \mathbf{A}(z)$ is the amplitude of the electric field of the light wave $\mathbf{E} = \mathbf{A}(z)\exp(ikz - i\omega t)$, ω and k are the frequency and wave vector of a wave that is normally incident on the layer, $q_a = \omega\varepsilon_a/(2cn_0)$ is the optical parameter, n_0 is the index of refraction for a normal wave, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$, where $\varepsilon_\parallel$ and $\varepsilon_\perp$ are parameters of the permittivity tensor at frequency ω, and $\mathbf{n}$ is the director defined through the angle of deviation φ from the undisturbed state of the director $\mathbf{n}_0$ by the relation $\mathbf{n} = (n_x, n_y) = (\cos\varphi, \sin\varphi)$.

In contrast to [5], the experiment described here involves examination of the angular incidence of a wave on an NLC layer and recording of the signal reflected from a solid surface. However, as will be shown below, this circumstance does not affect the generality of the ensuing arguments.

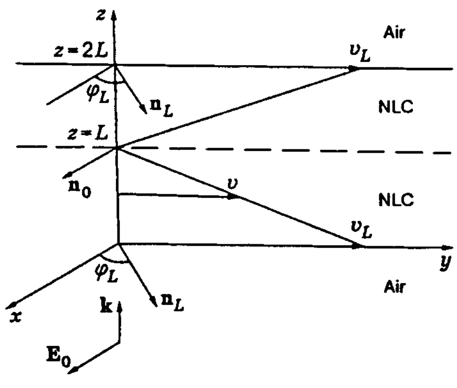

Fig. 8. Geometry of the problem equivalent to the conditions of the experiment.

From the solution of the hydrodynamic equations in [5], it follows that in our case, φ is given by the exponential relation

$$\varphi = 0.5\pi[1 - \exp(-\mu z)], \tag{5}$$

where $\mu^2 = (\alpha_2\alpha_3)^{1/2}w/K$, α_2 and α_3 are the second and third Leslie viscosity coefficients, K is the averaged Frank constant [Eq. (5) was derived in a one-constant approximation], and w is the gradient of the NLC velocity in the layer. All of the calculations in [5] were performed on the assumption that the velocity gradient was constant across the layer.

Before further discussion, let us evaluate the quantities q_a and μ for mixture 1282, used in the experiment. The optical parameter $q_a \approx 2.8 \cdot 10^{-4}$ cm^{-1} or $\lambda_a = 2\pi/q_a \approx 2.8$ μm ($\epsilon_a = 0.8$, $\lambda_{\rm inc} = 0.6$ μm, and $n_0 = 1.54$). To determine μ, it is necessary to estimate w. As is known [6], the tangential component of the tensor of viscous stresses (or the frictional force acting on a unit area of the plate in the flow) is given by the formula

$$\tau = 0.332(\eta_{\rm air}\rho_{\rm air}U_\infty^3/y)^{1/2}, \tag{6}$$

where $\eta_{\rm air}$ and $\rho_{\rm air}$ are the viscosity and density of air, and y is the distance along the flow from the edge of the plate to the point at which the measurement is made. The value of y is fairly large so that the condition $L \ll \delta$ is satisfied. Indeed, δ is the thickness of the boundary layer, given by the relation [6] $\delta = 1.72(y\eta_{\rm air}/\rho_{\rm air}U_\infty)^{1/2} \approx 600$ μm, and the characteristic thickness of the NLC layer in the experiment $L \leqslant 10$ μm.

Using the equality of the tangential components of the viscous-stress tensor, for the NLC–air interface we can write $\tau\big|_{z=L} = \tau_{\rm NLC}\big|_{z=L} = \eta_{\rm NLC}w\big|_{z=L} \approx \tau\big|_{z=0}$. Hence, we have

$$w = \tau/\eta_{\rm NLC}. \tag{7}$$

Substituting the parameters of the experiment [$U_\infty \approx 500$ cm/sec, $y \approx 2.5$ cm, $\eta_{\rm NLC} \approx 0.08$ g/(cm·sec), $\eta_{\rm air} = 2 \cdot 10^{-4}$ g/(cm · sec)] into (6) and (7), we obtain $\tau \approx 0.13$ Pa and $w \approx 16$ sec^{-1}. Knowing the value of w, we find the parameter $\mu = 447$ cm^{-1} or $\lambda_g = 1/\mu = 22$ μm. Thus, mixture 1282 satisfies the condition $\lambda_a \ll \lambda_g$.

This approximation makes it possible to write the Maxwell equations (4) in the adiabatic Mogen approximation, i.e., to ignore the dependence of the director $\mathbf{n}$ on the z coordinate. When we compare the solution of (4) in the Mogen approximation to the exact numerical solution for the case examined in [5], we obtain agreement of the order of 8% up to the value $\mu L = 0.75$. Such agreement is fully consistent with the conditions used in our investigation.

Figure 8 shows the geometry of the problem that is equivalent to the conditions of the experiment. It is taken into account that the wave is incident on the free surface of the NLC and passes through the specimen once more after it is reflected from the solid boundary. The solution is not difficult to write out in the given approximation. With allowance for the boundary conditions (Fig. 8), as the wave leaves the specimen its amplitude is $A_y = -A_0[1 - \exp(iq_a z)] \sin\varphi \cos\varphi$, where A_0 is the amplitude of the incident wave. The square brackets enclose the phase factor, which depends explicitly on the z coordinate. If we use the same approximation to examine the angular incidence of a wave on a plane–parallel layer, this factor changes form and becomes dependent on the y coordinate and the angle of incidence $\alpha_{\rm inc}$. However, this dependence will not affect subsequent calculations, and, hence, we will designate the phase factor as a certain function $\psi(z, y, \alpha_{\rm inc})$. In the experiment, we measure H/h the optical response at the exit from the layer ($z = 2L$). The relative intensity of the y-component of the wave has the form

$$H = A_y A_y^*/A_0^2. \tag{8}$$

In the absence of air flow, the normalizing signal h, determined by thermal fluctuations of the director has the form

$$h = a_y a_y^*/A_0^2, \tag{9}$$

where a_y is the fluctuation amplitude of the y-component of the wave. Writing H/h with allowance for (8) and (9), we eliminate the phase factors ψ and obtain

$$H/h = \langle \sin^2 2(\varphi_L + \varphi_f)\rangle / \langle \sin^2 2\varphi_f\rangle.$$

Here φ_f is the fluctuation complement for the angle of orientation of the director of the NLC. The angle brackets denote averaging over time. If we assume that $\varphi_f \ll \varphi_L$, we ultimately obtain

$$H/h = 0.25(\sin^2 2\varphi_L)/\langle\varphi_f^2\rangle + \cos^2 2\varphi_L. \tag{10}$$

We now examine the resulting relations in a linear approximation ($\varphi_L \ll 1$). Then (10) takes the form

$$H/h - 1 = \varphi_L^2/\langle\varphi_f^2\rangle. \tag{11}$$

The standard deviation of the position of the director from its mean position for thermal fluctuations [7] is written in the form

$$\langle\varphi_f^2\rangle \sim K_{\rm B}T/Kq_z^2 \sim L^2 K_{\rm B}T/K. \tag{12}$$

Here $K_{\rm B}$ is the Boltzmann constant, T is the temperature, and q_z is a transverse Fourier harmonic, which in our case is proportional to $1/L$. Substituting φ_L from (5) and $\langle\varphi_f^2\rangle$ from (12) into (11), we obtain

$$H/h - 1 \sim \mu^2 \sim w. \tag{13}$$

In turn, the parameter w is proportional to the tangential component of the viscous-stress tensor τ from (7). The quantity used in (7) to describe the viscosity of the NLC on the surface of the layer in the chosen geometry is determined through the angle φ_L [5]:

$$\eta_{\rm NLC} = 0.5[(\alpha_3 + \alpha_6)\sin^2\varphi_L + \alpha_4], \tag{14}$$

where α_3, α_4, and α_6 are Leslie viscosity parameters. Substituting (7) and (14) into (13) and assuming that φ_L is small, we finally obtain

$$H/h - 1 \sim w = 2\tau/\alpha_4. \tag{15}$$

It is apparent from this that, first, the optical response is independent of the thickness of the layer L and, second, it is proportional to the surface tension (3).

Relation (15) was obtained in the approximation $\varphi_L \ll 1$. In accordance with (5), the maximum deviation of the director $\mathbf{n}$ from its undisturbed state $\mathbf{n}_0$ is realized on the surface $z = L$ and in our case takes the value $\varphi\big|_{z=L} = \varphi_L = 0.5\pi[1 - \exp(-\mu L)] \approx 0.6$ rad. However, the dependence of $H/h - 1$ on τ remains linear (within the limits of accuracy of the experiment).

The first result is very important, since in many similar methods (see [8] and its bibliography) involving the exposure of a layer of an isotropic liquid to an air flow it is essential to know the thickness of the layer.

The proportionality factors relating the optical response to the sought parameters of the air flow were not calculated above, mainly because it is difficult to achieve completely uniform orientations of the NLC on the surface not only for two different specimens but also for one given substrate within the "spot" occupied by the crystal.

Thus, within the framework of the approximations used in this investigation, a completely acceptable level of agreement between theory and experiment has been obtained. The method proposed here has certain advantages over the methods described in [1–4], where cholesteric liquid crystals were used. First, the thickness of the liquid-crystal coating does not affect the signal during the "working interval" ($t_2 - t_1$ in Fig. 6) within the range of air-flow velocities examined above. Second, the response is not appreciably affected by the angle of observation, since the method employs the ratio of two signals, each of which has the same dependence on the angle of observation.

Among the shortcomings of the proposed method is the need for preliminary calibration. As can be seen from Fig. 6, the position of the "plateau" can change from specimen to specimen, affecting the value of the proportionality factor in (3). However, a more detailed study of this phenomenon will likely yield a simple solution that will eliminate the need for calibration measurements. The need to periodically turn off the flow in order to measure h is not especially burdensome if such a procedure is performed in an intermittent-type wind tunnel or in full-scale experiments.

REFERENCES

1. E. J. Klein, "Liquid crystals in aerodynamic testing," *Astronaut. Aeronaut.*, **6**, 1–8 (1968).
2. G. M. Zharkova, "Use of liquid crystals to visualize flows in an aerodynamic experiment," in: *Aerophysics Investigations* [in Russian], Inst. of Theor. and Appl. Mech., Sib. Div., Acad. of Sci. of the USSR (1972), p. 38.
3. D. C. Reda, J. J. Muratore, and J. T. Heineck, "Time and flow-direction responses of shear-stress-sensitive liquid crystal coatings," *AIAA J.*, **32**, No. 4 (1994).
4. D. C. Reda and J. J. Muratore, "Measurement of surface shear stress vectors using liquid crystal coatings," *AIAA J.*, **32**, No. 8, 1576–1582 (1994).
5. G. M. Zharkova and S. I. Trashkeev, "Shear flow of a nematic liquid crystal," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 5, 60–68 (1994).
6. L. G. Loitsyanskii, *Mechanics of Liquids and Gases*, Pergamon Press, Oxford-New York (1966).
7. P. de Gen, *Physics of Liquid Crystals* [Russian translation], Mir, Moscow (1977).
8. A. I. Maksimov, A. A. Pavlov, and A. M. Shevchenco, "Development of the friction measurement technique for supersonic gradient flows," in: *Proc. Int. Conf. Methods of Aerophys. Research.*, Vol. 2, Novosibirsk (1994), pp. 172–177.